PID: ______________________________

Last Name, First Name: ______________________________

Section: ______________________________

Midterm II
Math 20E, UCSD, Winter 2018
Thursday, March 1st, 3:30pm-4:50pm
Instructor: Eddie Aamari

- Write your PID, Name and Section in the spaces provided above.
- Do not unstaple the pages.
- Write your solutions clearly in the spaces provided.
- Answers written outside the answer boxes will not be graded.
- No calculators or other electronic devices are allowed during this exam.
- Put away (and silence!) your cell phone and other devices that can be used for communication or can access the Internet.
- Show all of your work; no credit will be given for unsupported answers.

DO NOT TURN PAGE UNTIL INSTRUCTED TO DO SO

Exercise	I	II	III	IV	V	Total
Points	5	5	5	5	5	25

Exercise I (5 points)

A helical wire follows the path $\mathbf{c}(t) = (3\cos(t), 3\sin(t), 4t)$ for $0 \leq t \leq 5\pi$. Its mass density m (mass per unit length) is given by $m(x, y, z) = 2z$. Find the mass of the wire.

Exercise II (5 points)

Let S be the parabolic surface given by $z = 9 - x^2 - y^2$ for $x^2 + y^2 \leq 9$.

1. Find a parametrization $\Phi : D \to S$. Be sure to specify the domain D.
2. Use your parametrization to find a normal vector to S at the point $(1, 2, 4)$.

Exercise III (5 points)

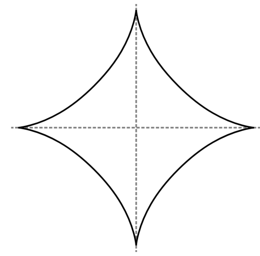
Let S be the part of the paraboloid $z = x^2 + y^2$ lying between the two planes $z = 1$ and $z = 4$. Compute the surface area of S.

Exercise IV (5 points)

Let S be the upper unit hemisphere (given by $x^2 + y^2 + z^2 = 1$ and $z \geq 0$), oriented using the outward normal. Let $\mathbf{F}(x, y, z) = (y, x, z)$ be a vector field. Compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

Exercise V (5 points)



The astroid curve $x^{2/3} + y^{2/3} = 1$ can be parametrized using

$$t \mapsto (\cos(t)^3, \sin(t)^3) \qquad \text{with } 0 \leq t \leq 2\pi.$$

Use Green's theorem to compute the area it encloses.

Helpful identities:

$$\cos(x)^2 = \tfrac{1}{2}(1 + \cos(2x)), \qquad \sin(x)^2 = \tfrac{1}{2}(1 - \cos(2x)), \qquad \sin(2x) = 2\sin(x)\cos(x).$$